T.D. 11 : Développements limités CORRECTION

1. Calculer les limites suivantes :

i) $\lim\limits_{x\to 0}\dfrac{\ln(\cos 3x)}{\sin^2 x} = \dfrac{-9}{2}$ car $\dfrac{\ln(\cos 3x)}{\sin^2 x} \underset{x\to 0}{\sim} \dfrac{-\dfrac{(3x)^2}{2}}{x^2}$

ii) $\lim\limits_{x\to 1}\dfrac{x^2-1}{e^{2x-1}-e^x} = 2e^{-1}$ car $\dfrac{x^2-1}{e^{2x-1}-e^x} \underset{x=1+h}{=} \dfrac{2h+h^2}{e^{1+2h}-e^{1+h}} = \dfrac{h(2+h)}{e\left(e^{2h}-e^h\right)} \underset{h\to 0}{\sim} \dfrac{2h}{e(2h-h)}$

iii) $\lim\limits_{x\to 1}\dfrac{\left(x^2-3x+2\right)\sin(x\pi)}{\ln\left(x^2-2x+2\right)} = \pi$ car

$$\frac{\left(x^2-3x+2\right)\sin(x\pi)}{\ln\left(x^2-2x+2\right)} \underset{x=1+h}{=} \frac{\left(-h+h^2\right)\sin\left(h\pi+\pi\right)}{\ln\left(1+h^2\right)} = \frac{\left(h-h^2\right)\sin\left(h\pi\right)}{\ln\left(1+h^2\right)} \underset{h\to 0}{\sim} \frac{h\times h\pi}{h^2}$$

iv) $\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{1}{x}}-e}{x} = \dfrac{-e}{2}$ car $\dfrac{(1+x)^{\frac{1}{x}}-e}{x} = \dfrac{e^{\frac{1}{x}\ln(1+x)}-e}{x} \underset{x\to 0}{\sim} \dfrac{e^{\left(1-\frac{x}{2}\right)}-e}{x} \underset{x\to 0}{\sim} \dfrac{e\left(1-\dfrac{x}{2}-1\right)}{x}$

2. Calculer le $DL_n(0)$ pour les expressions suivantes :

i) $\dfrac{1}{\sqrt{1-x}} \underset{x\to 0}{=} 1+\dfrac{x}{2}+\dfrac{3}{8}x^2+\dfrac{5}{16}x^3+\dfrac{35}{128}x^4+\dfrac{63}{256}x^5+o\left(x^5\right)$

ii) $\dfrac{1}{\sin x}-\dfrac{1}{x} \underset{x\to 0}{=} \dfrac{1}{6}x+o(x)$

iii) $2\ \mathrm{Arctan}\left(e^x\right) \underset{x\to 0}{=} \dfrac{\pi}{2}+x-\dfrac{1}{6}x^3+o\left(x^3\right)$

iv) $\ln^2(1+x) \underset{x\to 0}{=} x^2-x^3+\dfrac{11}{12}x^4+o\left(x^4\right)$